Name:

Math 10a

September 25, 2014

Quiz #3

1. Let $f(x) = \sqrt{x^2(1-x^2)}$

   (a) (1 point) For what values of x is f defined?

   f is defined when the expression under the radical isn't negative, so for $\boxed{|x| \leq 1}$.

   (b) (2 points) What are the largest and smallest values of $f(x)$ on its domain of definition?

   The domain we're looking at is $[-1, 1]$, so the endpoints are ± 1. The critical points can be determined by differentiating the function:

$$f'(x) = \frac{1}{2}(x^2(1-x^2))^{-1/2}(2x(1-x^2) - 2x^3) = \frac{x - 2x^3}{\sqrt{x^2(1-x^2)}}.$$

   This is zero precisely when the numerator is zero, so

$$x - 2x^3 = 0 \Rightarrow x(1 - 2x^2) = 0 \Rightarrow x = 0, \pm\frac{1}{\sqrt{2}}.$$

   Checking these values:

$$\begin{aligned} f(\pm 1) &= 0 \\ f\left(\pm\frac{1}{\sqrt{2}}\right) &= \frac{1}{2} \\ f(0) &= 0. \end{aligned}$$

   So the largest value is $\frac{1}{2}$ and the smallest value is 0.

2. (5 points) Graph
$$\frac{(x-2)^2}{x+2}.$$
Indicate (if any) critical points, inflection points, asymptotes, and intercepts.

To find critical points we compute the derivative

$$(x-2)^2(x+2)^{-1} \overset{\frac{d}{dx}}{\to} 2(x-2)(x+2)^{-1} - (x-2)^2(x+2)^{-2}$$

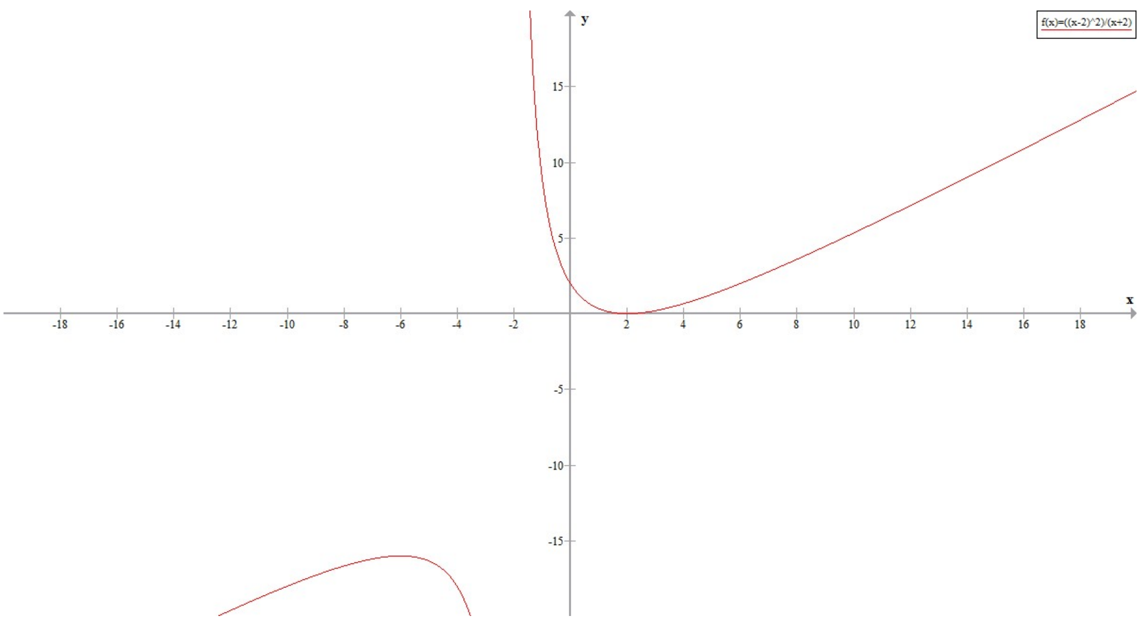
$$= \frac{2(x-2)}{x+2} - \frac{(x-2)^2}{(x+2)^2} = \frac{2(x-2)(x+2)-(x-2)^2}{(x+2)^2} = \frac{x^2+4x-12}{(x+2)^2} = \frac{(x+6)(x-2)}{(x+2)^2}$$

so critical points at $(-6,-16)$ and $(2,0)$. The second derviative is therefore

$$(x^2+4x-12)(x+2)^{-2} \overset{\frac{d}{dx}}{\to} (2x+4)(x+2)^{-2} - 2(x^2+4x-12)(x+2)^{-3}$$

$$= \frac{2x+4}{(x+2)^2} + \frac{-2x^2-8x+24}{(x+2)^3} = \frac{(2x+4)(x+2)-2x^2-8x+24}{(x+2)^3} = \frac{32}{(x+2)^3}$$

which is never 0. So there are no inflection points. There's a horizontal asymptote at $x=-2$ and intercepts at $(2,0)$ and $(0,2)$.



3. (1 point) A population of bacteria P (measured in mg) changes with time t according to the equation

$$\frac{dP}{dt} = k(M - P)$$

for some positive constants k and M. If, initially, there are 10 mg of bacteria and then the researcher returns later to find only 6 mg of bacteria, what can you say about M?

The derivative must be negative for the population to decrease, so $\boxed{M < 10}$.

4. (1 point) The growth rate G of an amoeba population is modeled by

$$G(t) = a_1 t - a_2 t^3$$

for positive constants a_1 and a_2 and $t > 0$. When is the amoeba population growing fastest?

G is the growth rate, so the amoeba population grows the fastest when G is the largest. We need to maximize G for $t > 0$, so we compute its derivative:

$$G'(t) = a_1 - 3a_2 t^2$$

which is 0 when $\boxed{t = \sqrt{\frac{a_1}{3a_2}}}$. This is a local max, so G must be maximized there.